

Cooper pairs at above-critical current region

Yongle Yu

Received: date / Accepted: date

Abstract It is generally believed that in a superconductor Cooper pairs are broken at above-critical current region, corresponding to the lost of superconductivity. We suggest that, under some circumstance, Cooper pairs could still exist above critical current, and that dissipation of the system is caused by the scattering of these pairs. The existence of Cooper pairs in this region can be revealed by investigating the temperature dependence of the electrical resistance.

Keywords critical current · cooper pairs · dissipation

1 Introduction

The formation of Cooper pairs in a superconductor is essential for superconductivity [1][2]. Cooper pairs are of bosonic nature, and it is believed that they generate a supercurrent in the same mechanism as helium atoms generate a superflow. It is clear that, once the cooper pairs in a superconductor are broken, superconductivity will be lost. An interesting question is that, can the system become dissipative without the breaking of cooper pairs?

From the view point of many-body physics, one can see there is a rather striking difference between the origin of Cooper pairs and the origin of superconductivity. Cooper pairs is due to some *attractive* interactions between fermions, while superconductivity, like superfluidity, has something to do with *repulsive* interactions between its composing bosons, *i.e.*, the Cooper pairs [3][4]. This difference implies that the physical

regime of Cooper pairs is not exactly the same as that of superconductivity. We suggest that, under some circumstances, Cooper pairs are not broken when one increases a supercurrent to above critical current. We shall also show that, at the above-critical current region, the electrical resistance, caused by the scattering of cooper pair, has a different temperature dependence from the case where the charge carriers of the current are fermions (electrons or holes). Thus the existence of Cooper pairs can be signified by the temperature behavior of the resistance.

Like the case of superfluidity [3][4][5], the dissipative behavior of a superconductor above a critical current I_c , can be explained naturally in terms of the many-body dispersion spectrum $E(I)$ of the charge carrier system ($E(I)$ is the lowest eigen energy at given current I). Beyond I_c , $E(I)$ is a monotonically increasing function of I (see Fig. 1), thus a current can continuously lose its energy and momentum to the environment, corresponding to a dissipative decay process. This is contrast to the case in the $I < I_c$ regime where the supercurrents are metastable states, corresponding to the local minima of the $E(I)$ curve, whose decay is prevented by the energy barriers among the minima.

The dissipation mechanism illustrated above does not involves breaking of Cooper pairs. Moreover, the low-lying eigen energy states below I_c can be Galileo boosted to generate the low-lying states above I_c [3][5][6]. The Galileo boost, also could being viewed as center-of-mass-mention transformation, does not modify the inner structure of the states such as pair correlations. Thus, the Cooper pairs, present in states below I_c , survive the boost and exist in the states above I_c [7].

In literature, Landau's criterion of a superconductor is generally used for the analysis of transitional regime, which determines a critical velocity to be Δ/p_F , where

Yongle Yu
Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan, 430071, China
E-mail: yongle.yu@gmail.com

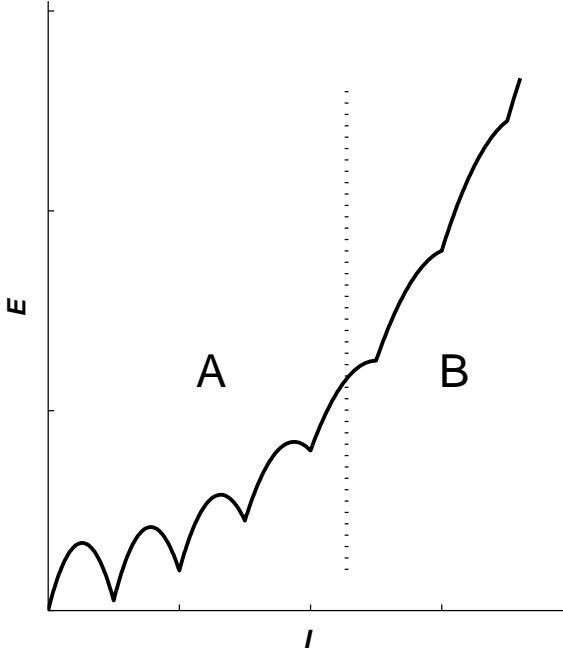


Fig. 1 A schematic plot of the many-body dispersion spectrum of the charge carrier system in a superconductor. Region A is the superconducting region where there are metastable current-carrying states corresponding to the local minima of the curve; Region B is dissipative without metastable states. The many-body eigen energy states at region A can be mapped to the states at region B by Galileo transformation.

Δ is the pairing gap and p_F is the Fermi momentum. Landau's criterion requires an exchange of a quantum, with a large momentum (the order of p_F) and with a certain energy, between the superconducting charges and its environment. However, this exchange process could be inhibited for that the spectrum of the environment may generally be incompatible to absorb such an unusual quantum.

In a superconductor, the self-induced magnetic field of the supercurrent could lead to loss of superconductivity by breaking cooper pairs, (*i.e.*, Silsbee effect [8]). In this case, there is an 'external' critical current determined by the critical magnetic field, and the Cooper pairs do not exist above critical current. However, Silsbee effect can be largely reduced by alignment of the currents and their geometries (see Fig. 2), thus one might be able to obtain an intrinsic critical supercurrent, like the case of superfluid ^4He , and reach a dissipative regime without breaking Cooper pairs. We shall assume the existence of Cooper pairs above critical current in follows.

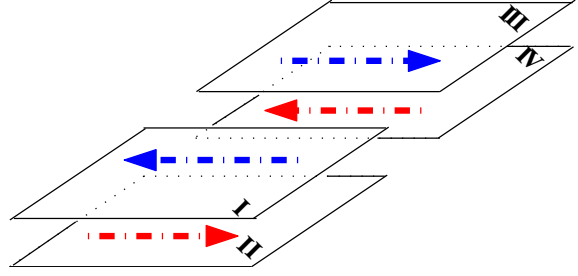


Fig. 2 To weaken the Silsbee effect, four thin superconducting films are used to carry the current. Film I is on top of film II, and film III on top of film IV. Film I is in the same plane of film III, and film II in the same plane of film IV. Current in the film I (blue arrow) points to the same direction as to the current in film IV (red arrow), while the currents in the film II (red arrow) and film IV (blue arrow) point to the opposite direction.

2 Analysis and Results

The electrical resistance in a superconductor above I_c and below T_c is mainly caused by the scattering properties of Cooper pairs with phonons. We shall discuss the temperature behavior of the resistance. For simplicity, we assume physical properties of the superconductor are isotropic. First, we consider the dispersion relation of a Cooper pair. The dispersion is linear at small momentum \mathbf{q} , *i.e.*, $\varepsilon(\mathbf{q}) \approx v|\mathbf{q}|$ (we approximate v by the critical velocity of the supercurrent v_c). At large \mathbf{q} , the energy of a Cooper pair is approximately $q^2/2m_c$ where m_c is the mass of a Cooper pair assumed to be $2m_e$ (m_e is the mass of an electron). A general form of dispersion

$$\varepsilon(\mathbf{q}) = \sqrt{v_c^2 q^2 + (q^2/4m_e)^2} \quad (1)$$

can be used for approximation for all \mathbf{q} values [9].

We consider the leading scattering process in which one phonon is adsorbed or emitted by a Cooper pair (see Fig 3). The total energy and momentum should be conserved,

$$\varepsilon(\mathbf{q}_1) = \varepsilon(\mathbf{q}_2) \pm \hbar c_s p \quad (2)$$

$$\mathbf{q}_1 = \mathbf{q}_2 \pm \mathbf{p} \quad (3)$$

Where \mathbf{q}_1 (\mathbf{q}_2) is the momentum of a cooper pair before (after) the scattering, \mathbf{p} is the momentum of the phonon, c_s is the sound velocity, and $+$ ($-$) corresponds to the emission (absorption) of the phonon.

Combining Eq. 1, Eq. 2 and Eq. 3, one gets,

$$\sqrt{v_c^2 q_1^2 + (q_1^2/4m_e)^2} = \sqrt{v_c^2 q_1^2 + (q_1^2/4m_e)^2} \pm \hbar c_s |\mathbf{q}_1 - \mathbf{q}_2| \quad (4)$$

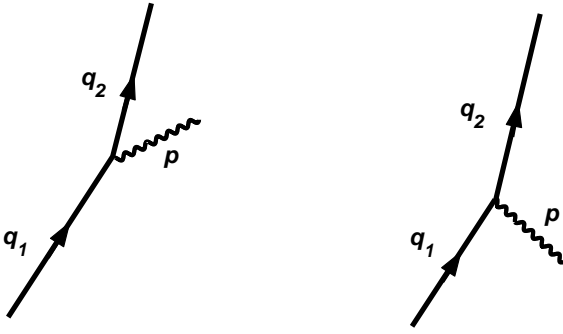


Fig. 3 One phonon scattering process of a Cooper pair. left (right) corresponds to the emission (absorption) of a phonon.

In most superconductors, v_c , can be estimated using critical current density [10], is roughly the orders of ten meters per second or below. Thus v_c is orders of magnitude smaller than c_s . One then can realize that unless the q_1 (q_2) in the phonon emission (absorption) process is roughly equal or larger than $2m_e c_s$, Eq. 4 can not be satisfied. for a Cooper pair with a momentum of $2m_e c_s$, the energy is roughly $m_e c_s^2$ and the corresponding temperature is $T^* = m_e c_s^2 / k_B$ where k_B is the Boltzmann constant. Thus at low temperature $T < T^*$, one-phonon scattering process is absent, and the scattering process of Cooper pairs must involve at least two phonons.

One can invoke Boltzmann transport equation to determine the temperature dependence of resistance of the Cooper pair system. However, unlike the case of a fermionic system where the existence of Fermi surface and Pauli blocking are essential, the analysis of the (bosonic) Cooper pair system can be simplified. At $T > T^*$, the one-phonon scattering process can bring the momentum of a Cooper pair (with an energy of $k_B T$ or less) to zero or to the opposite direction, thus effectively causing the dissipation of the current. The probability of one phonon process in low temperature is proportional to T (see, *e.g.*, [11]), thus the resistance R is linear in T . At $T < T^*$, one can find that probability of two-phonons process is proportional to T^2 , thus $R \propto T^2$. In metals where temperature dependence of resistance is determined by electron phonon scattering, the power law of $R(T)$ at low temperature is different, with an exponent being 5 (see, *e.g.*, [11][12][13]). Thus one can distinguish between Cooper pairs and electrons by checking the powering law of $R(T)$ in the above critical current regime.

In the so called strange metal phase of some high- T_c cuprates, the resistance has also a linear temperature dependence. One might wildly speculate that this linear behavior could be caused by the scattering of Cooper pairs. In order for this speculation to be valid, Cooper

pairs shall exist above T_c in some systems. An example is that cold Fermi atom gases which can be tuned to go through BCS-BEC crossover. In the BEC side, the binding energy of Cooper pairs (or molecules) can be orders of magnitude larger than superfluid transition temperature T_c , since T_c can be made small by decreasing the density of atom gases.

3 Conclusions

we suggest that, under some circumstance, Cooper pairs could exist in a superconductor above critical current, and the system has a different power law of $R(T)$ from the case where electrons are current carriers.

Acknowledgements This work was supported by Chinese NSF Grant No. 11075201.

References

1. L. N. Cooper, *Phys. Rev.* **104** 1189 (1956).
2. J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108** 1175 (1957).
3. Y. Yu, *Ann. Phys.* **323** 2367 (2008); Y. Yu, arXiv:cond-mat/0609712.
4. A. J. Leggett, *Rev. Mod. Phys.* **73** 307 (2001).
5. F. Bloch, *Phys. Rev. A* **7**, 2187 (1973).
6. In superconductors, Galileo invariance is not exact but is a good approximation when the center of mass motion is not large.
7. there might be some possibility that the dispersion energy of normal state (without pairing) could be smaller than the dispersion energy of pairing state within a current regime somewhere between $I_c = 0$ and the first supercurrent, we ignore this case here.
8. F. B. Silsbee, *Journal of the Washington Academy of Sciences* **6**, 79 (1916).
9. for an analogy with a Bose system, see X. G. wen, *Quantum field Theory of Many-Body Systems* (Oxford University Press, London, 2004).
10. We obtain v_c using $j_c = n_c * (2e) * v_c$, where j_c is the critical current density, n_c is the density of Cooper pairs, $2e$ is the charge of a Cooper pair. We assume that all conducting electrons are paired up, thus $n_c = n_e/2$, where n_e is the density of electrons.
11. N. F. Mott and H. Jones, *Theory of Metals and Alloys* (Oxford University Press, London, 1936).
12. A. Eling and J. S. Schilling, *J. Phys. F: Metal Phys.* **11** 623 (1981).
13. G. J. Van den Berg, *Physica* **14** 111 (1948).